Entry effects in the open thermosyphon

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In this paper flow-visualization techniques are used to study the flow instability which occurs at the orifice in the free convection open thermosyphon. The influence of the shape of the orifice on the measured heat transfer of the system is also studied under both laminar and turbulent conditions. The dominant mode of penetration of the hot stream by the cold fluid entering at the base of the reservoir is found to instigate a mixing region at the orifice. In turbulent flow this spreads into and eventually fills the tube as the Rayleigh number is increased. In such circumstances a sharp-edged orifice appears to give better overall heat transfer than a rounded orifice.

Introduction

During the last 10 years the free convection open thermosyphon has been the subject of a great deal of research, both theoretical and experimental, designed to gain fundamental knowledge concerning the several flow régimes which occur in the device and the resulting heat-transfer rates. This work arose from the projected applications of the thermosyphon to gas turbine rotor blade cooling and the extraction of heat from nuclear reactors. The geometry of the device is such that a heated cylinder closed at its lower end opens into a reservoir of cool fluid. The heated fluid rises over the walls of the cylinder through the action of convective forces and is replaced by a returning core of cool fluid from the reservoir.

Mainly as a result of the contributions of Lighthill (1953), Martin (1955), Hartnett & Welsh (1957), Leslie & Martin (1959), Bayley, Milne & Stoddart (1961) and Bayley & Czekanski (1963) the unusual heat-transfer characteristics of the vertical system, particularly under turbulent conditions, are now reasonably well understood. Further studies by Hartnett, Welsh & Larsen (1959), Leslie (1959), and Martin (1959), have shed light on the effect of tilting the thermosyphon, thus inducing a lateral acceleration which to some extent simulates the Coriolis accelerations arising from rotation in gas turbine rotor-blade cavities. Tests under such conditions have been carried out by Ellerbrock (1951).

At the present time little is known of the region of origin of the cool fluid, and the method by which it enters and reaches the central core of the thermosyphon tube from the reservoir mounted above, having regard to the fact that heated fluid adjacent to the tube wall is simultaneously being discharged into the reservoir. It is reasonable to suppose that the mode of entry is influenced not only by the Rayleigh number, but also by (*a*) the inclination, in the case of the stationary tube, to the vertical, and (*b*) the geometry of the junction between tube and reservoir.

The shape of the orifice may therefore also be expected to have some effect on the heat-transfer characteristics of the system, since the extent to which the cool fluid comes into contact with the rising hot stream will determine the temperature of the former when it reaches the core of the tube. This has generally been used as one of the reference temperatures when computing values of the various dimensionless groups, though several workers have experienced great difficulty in its measurement because of extensive random fluctuations under certain flow conditions. Such fluctuations have usually been attributed to general fluid turbulence, an explanation which takes little or no account of the fact that they have also been observed in laminar flow.

The experimental work to be described is in two parts. The first is a flow-visualization study of entry phenomena, for both the vertical and the inclined cases, using convective fluids of both high and low viscosities, in conjunction with appropriate tracer techniques. Since instrumentation requirements conflicted with the needs of flow visualization, no measurements were made. The second part of the work is quantitative, and is concerned with the effects on the heat transfer of changes in the shape of the entry orifice for the vertical system under both laminar and turbulent flow conditions.

Experimental techniques

The apparatus used for the flow-visualization studies consisted of a thin glass tube of circular section, 1 in. in internal diameter and $7\frac{1}{2}$ in. long, (and therefore of length-radius ratio 15), closed at one end. The open end, which formed a sharp-edged orifice, was connected to the base of a reservoir, the latter being completely fabricated from Perspex. Araldite was used to make the connexion. The reservoir took the form of a box 8 in. square and 6 in. in height. A plate window 3 by 4 in. was incorporated in one side of the reservoir for photographic purposes, and arrangements were made for tilting the apparatus. The tube was electrically heated by a 2 ft. length of 32 s.w.g. nichrome resistance wire wound in the form of a helix around the outside of the tube. The pitch of the wire, which was sufficiently great to permit visual observation of the contained fluid, can be gauged from figure 2, plate 1, and figure 3, plate 2. The convective fluids used were golden syrup, rape-seed oil, and water. The thermal capacity of the reservoir fluid was sufficiently great to render the incorporation of a cooling coil unnecessary.

Since fluid velocities in free convection are generally low, flow-visualization techniques which depend on the use of a tracer fluid are often unsatisfactory because of the tendency of the trace to settle or diffuse before it has served its purpose, or while it is doing so. The path of the fluid particles is thus masked. This tendency can be minimized by choosing a tracer whose density corresponds as closely as possible to that of the convective fluid. Even so, where the latter is of high viscosity, so that flow velocities are even smaller, the time available for settling is increased. In such instances conventional tracer fluids cannot readily be utilized.

When water was used as the convective fluid, a solution of printers' 'process white' in water was found to be very satisfactory in that it did not readily settle. Furthermore, it photographed well, as can be seen from figures 2 and 3. The figures also indicate the method of introduction of the tracer fluid into the water at the base of the reservoir, via a gravity feed from an external source to one or two glass tubes. These are drawn down at their extremities to form small-bore orifices, whose principal axes lie parallel to the base of the reservoir, for reasons to be discussed below.

The flow was observed in the case of the viscous fluids (golden syrup and rape-seed oil) by noting the motion of very small air bubbles which had become entrained by vigorous agitation of the convective fluid in the reservoir. The bubbles are retained in their initial positions, for a sufficient period of time for observations to be made, by the high viscosity of the fluid. Both golden syrup and rape-seed oil are strongly coloured, and this technique, while permitting observation, did not lend itself to satisfactory photography.

The rig used to determine the effect of different shapes of entry orifice on the heat-transfer performance of the vertical system was similar to that described by Martin (1955), which incorporated a rounded orifice of $\frac{1}{4}$ in. radius in a 2 in. diameter tube. His investigation covered a series of length-radius ratios by utilizing tubes of different length, but in the present tests only one tube, 15 in. long and 2 in. in diameter, was used. One end of the tube, which was $1\frac{3}{8}$ in. thick, was counterbored to allow fitment of various orifice inserts. By this technique the $\frac{1}{4}$ in. radius rounded orifice originally incorporated to confirm the results obtained by Martin (1955, 1959) was replaced, first by a rounded orifice of $\frac{7}{8}$ in. radius, and afterwards by a sharp-edged orifice with a 90° included angle.

Rape-seed oil and water were used as convective fluids during these tests, which were all conducted at uniform tube-wall temperature. This was taken as the reference temperature for the determination of property values when evaluating the appropriate dimensionless parameters. Information is presented in the conventional form of Nusselt number versus Rayleigh number on double-logarithmic scales. For ease of reference these scales are duplicated, one being based on tube radius and the other on tube length, denoted by subscripts r and L, respectively.

Flow-visualization studies

As indicated in the introduction, little consideration has so far been given to the flow patterns in the region of the orifice. Certainly in the theoretical studies which have been made, it has always been assumed that the thermosyphon tube was connected to, and continuously replenished by, an infinitely large reservoir of fluid at uniform temperature. Even so, the very nature of the device suggests the likelihood of heat transfer between the hot and cold fluid streams outside the tube, and therefore in the reservoir. The extent of such heat-transfer leads naturally and particularly to the question of the origin of, and the path followed by, the incoming fluid. That followed by the hot stream is perhaps more readily perceived.

Preliminary tests, using both tracer techniques, showed the existence of an orifice flow pattern basic to both laminar and turbulent flow. This is illustrated in figure 1. Necking of the heated annulus takes place after it has left the tube, during its vertical passage through the layers of cool fluid in the reservoir. Holt, Skipper & Saunders (1961) refer to a similar chimney of fluid rising from a heater immersed in a highly viscous oil. Entry fluid (of greatest density) is drawn from layers adjacent to the floor of the reservoir, to approach the orifice radially from all directions. The two fluid streams collide, and an instability results from the penetration of the heated annulus by the cool fluid before it can enter the thermosyphon. The mode of penetration may take one of two forms. These are described below. In both cases, the cool fluid which does not gain entry to the thermosyphon is carried upwards with the hot stream, as shown in figure 1.

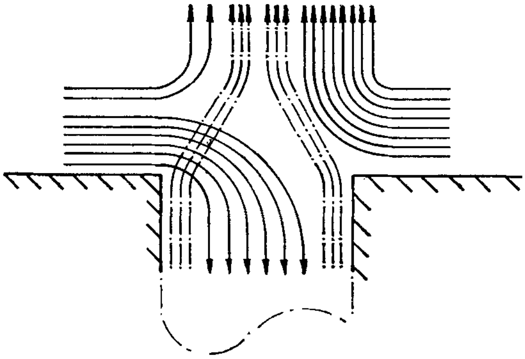


FIGURE 1. Basic orifice flow pattern. ———, Cool fluid; —·—·, hot fluid.

(*a*) *Laminar flow*

Further detailed tests using rape-seed oil and the more viscous golden syrup indicated that in laminar flow the mode of penetration is governed by the heat-transfer rate, and therefore, for a given tube, by the Rayleigh number. At very low heat fluxes, especially with golden syrup, the annular curtain of hot fluid immediately above the orifice parts locally at one or more positions round its circumference. Through the approximately egg-shaped holes thus formed the cool fluid gains access to the thermosyphon core. Although the orientation and size of the holes were observed over lengthy periods to change in random fashion, the total area presented to the incoming flow appeared to be constant, and consistent with continuity requirements in steady flow.

This method of penetration appears to involve little or no mixing of the fluid streams. The temperature of the entering fluid may therefore be expected to remain constant. Martin (1955) found that in laminar flow this was the case only for the impeded similarity and non-similarity régimes associated with Nusselt numbers (based on radius) below 2, corresponding in all probability to the very low heat fluxes mentioned above. There are therefore reasonable grounds for believing that this mode of penetration is confined to laminar flow in which the boundary layer, so to speak, fills the tube.

For Nusselt numbers above 2, Martin (1955) first observed regular periodic oscillations of fluid entry temperature, corresponding to the onset of laminar boundary-layer flow. When this régime had become established, the oscillations were succeeded by irregular random fluctuations in the entry temperature. These findings are also in accord with a change in the mode of penetration of the entry fluid observed in the present studies as the heat flux was increased.

The second method of penetration differs from the first in that there are regions around the circumference where the entry fluid mixes with the emerging heated annulus and carries it back into the tube. Thus the flow direction of some of the hot fluid is reversed. Meanwhile round the remaining portions of the circumference the rejected entry fluid is, as before, swept upwards by the hot fluid. The positions of the regions of eddying and mixing change frequently and suddenly without any apparent reason. The instabilities resulting from this random mixing would appear to be responsible for the fluctuations in fluid entry temperature.

Of particular interest is the fact that for both viscous fluids the mixing and local turbulence in the tube are always confined to a shallow region extending not more than $\frac{1}{2}$ in. below the orifice. In the remainder of the tube the flow is laminar and steady, notwithstanding the entry disturbance.

(*b*) *Turbulent flow*

Previous work by Martin (1955) has shown that the substitution of water as convective fluid, with its lower viscosity, generally brings about an early transition to turbulent flow in the open thermospyhon. It is, therefore, not surprising that in the flow-visualization tests using water the second mode of penetration occurs even at the lowest heat fluxes. It is found that with increasing heat flux, the shallow mixing region initially near the orifice extends further and further into the tube. This extension progressively reduces the length of the tube occupied by the boundary layer, and corresponds to the development of fully-mixed flow, which ultimately occupies the whole tube.

The photographs of figure 2 indicate the second form of penetration of the entry fluid, and the characteristic instability. In figure 2(*a*) fluid on the right-hand side is seen to enter the tube in the region of the wall, while diametrically opposed fluid on the left-hand side, after approaching the orifice, is rejected in a surge of rising hot fluid. A few seconds later the situation is reversed, as shown in figure 2(*b*), in which the tendency for the direction of flow of the hot stream to be reversed by the entering fluid may also be observed. The boundaries of the mixing region proved difficult to determine by visual observation because of the unsteady nature of the flow situation; in this connexion Saunders (1936) has demonstrated the sensitivity of free-convection flow in general to small external disturbances. This must be particularly true of entry effects in the open thermosyphon.

In the light of Martin's (1959) suggestion that the effect of tilting the open thermosyphon is, under turbulent conditions, to clear the already established fully-mixed flow (and by implication, therefore, the factors which promote it) by aiding the transfer of fluid to the hot stream, visual observations of the flow were made with the tube and reservoir inclined over a range of angles up to 45°. The random selection of entry fluid is found to become steadily less marked as the tube is tilted, and appears to be eliminated for tilting angles greater than about

10°. Cool fluid then enters along the so-called 'trailing edge', while hot fluid escapes freely at the 'leading edge' into the reservoir. The orifice instability present when the tube is vertical thus disappears, and so, in consequence, does the downward-spreading fully-mixed flow régime which it initiates. As Martin (1959) suggested, this leaves the way open for the establishment of the boundary-layer régime, with increased heat transfer, over the whole tube.

Photographs illustrating these effects are shown in figure 3 for an inclination of 45°. In figure 3 (*a*), with small heat transfer, the trace shows that cool fluid enters the tube in laminar flow towards the trailing edge, but nearer the core than when the tube is vertical, as in figure 2. It approaches the wall as it penetrates the tube, subsequently to be drawn into the boundary layer. The rising streaks of trace fluid, which coincide with the pitch of the heating coil, and are due to hot spots at the wall, indicate the path of the fluid around the circumference towards the leading edge of the tube. The final egress of the fluid from the orifice is also visible. The flow in this case is unbroken by the presence of mixing at the orifice. It is sufficiently clear to render unnecessary the introduction of trace fluid through the small bore orifice shown on the left-hand side of figure 2, which was therefore removed.

Figure 3 (*b*) shows the effect of increased heat transfer. Before the descending filament reaches the wall it breaks down to the wavelike motion of turbulence. This contributes to the establishment of fully-mixed flow, which, in the vertical tube, is propagated from the orifice. When the tube is tilted, figure 3 (*b*) suggests that it may develop from the region near the closed end, as there is visible evidence of considerable mixing in this region. But the hot fluid still escapes from the tube along the leading edge. The waves developed in the entry filament, and the suggestion of vortex formation prior to the sudden breakdown to random turbulence, correspond to the description given by Eckert, Hartnett & Irvine (1960).

The orifice instability discussed above for the vertical tube probably becomes steadily less marked if its length-radius ratio is reduced, because of the decreasing tendency of the heated annulus to neck. Ultimately cool fluid should gain unrestricted entry to the tube from the region above the orifice, in which case its temperature should remain steady, as found by Bayley *et al.* (1961) using mercury in tubes of length-radius ratio in the range from 1·25 to 2·58.

Lastly, it may be argued that when the entry fluid undergoes an appreciable rise in temperature due to mixing before it enters the tube, the well-established use of the orifice mean entry temperature in computing dimensionless groups can lead to deceptively high Nusselt numbers and correspondingly low Rayleigh numbers. On the contention that the effect of the thermosyphon should be considered as a whole, a more realistic temperature in such cases would be that of the fluid at the bottom of the reservoir remote from the orifice. On the other hand, it has generally been argued in the past that only the heat-transfer characteristics of the thermosyphon (without the reservoir) are under consideration, and therefore the orifice mean entry temperature is appropriate. That there are convincing arguments on both sides makes the question difficult to resolve, but in the test results described below, the orifice mean entry temperature is used so as to accord with previous practice.

Effect of orifice shape on heat transfer

(*a*) *Laminar flow*

A preliminary test using a $\frac{1}{4}$ in. radius rounded orifice gave satisfactory agreement with Martin's (1959) experimental data for rape-seed oil in a vertical tube of length-radius ratio 15, from which he deduced that throughout the test the flow remained of the the laminar boundary-layer type.

The test results when the $\frac{3}{8}$ in. rounded orifice and the 90° included angle sharp-edged orifice were substituted are shown in figure 4. Two experimental curves may be distinguished, both of uniform slope. That of the upper curve (at low Rayleigh numbers) is 0·27, and corresponds to the $\frac{3}{8}$ in. rounded orifice.

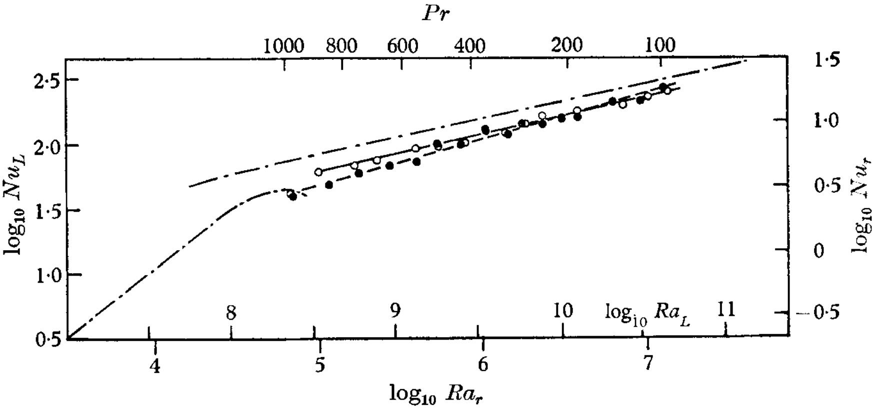
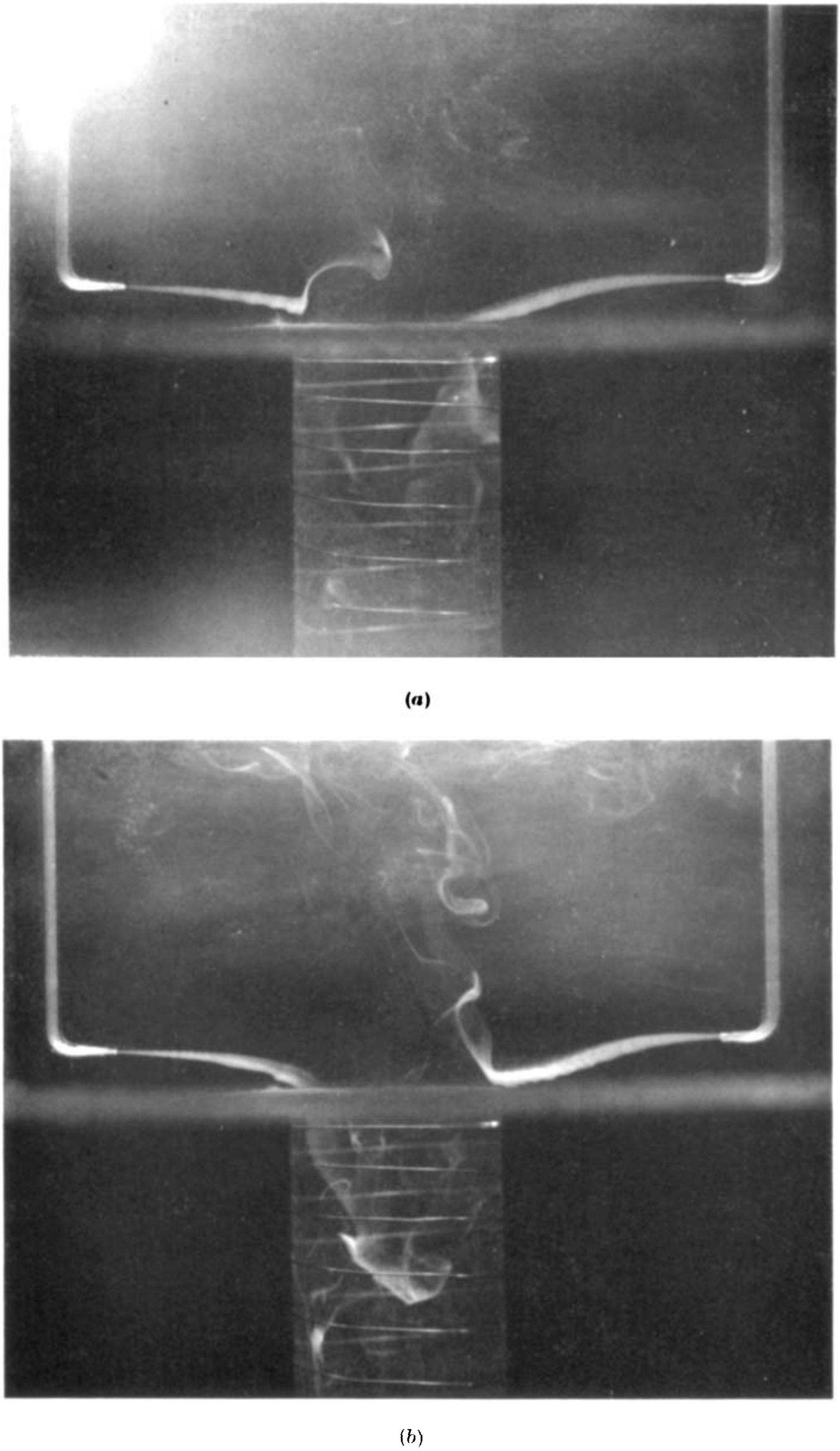


FIGURE 4. Experimental heat-transfer rates in laminar flow for rounded and sharp-edged orifices. ○, Results for $\frac{3}{8}$ in. radius orifice; slope = 0·27. ●, Results for sharp-edged orifice; slope = 0·36. —·—·—, Lighthill's theoretical laminar curve, $Pr = \infty$.

The lower curve refers to the sharp-edged orifice and has a slope of 0·36. The curves intersect when $Ra_L = 10^{10 \cdot 1}$ so that for higher Rayleigh numbers their positions are reversed. Although for clarity the confirmatory test results using a $\frac{1}{4}$ in. rounded orifice are omitted, they in fact occupy an intermediate position with respect to the curves shown over most of the range. For purposes of comparison, Lighthill's (1953) theoretical curve for laminar boundary-layer flow and infinite fluid Prandtl number has been included; it lies above the present experimental data.

This discrepancy was attributed by Martin (1959) to under-development of the laminar boundary-layer régime in tubes of relatively small length-radius ratio, the flow being still to some extent characteristic of impeded non-similarity laminar flow, with lower heat transfer. This would imply some intermediate mode of penetration of the entry fluid, relative to those described in the previous section, with possibly a decreasing bias towards flow reversal of part of the heated fluid at the lower Rayleigh numbers.

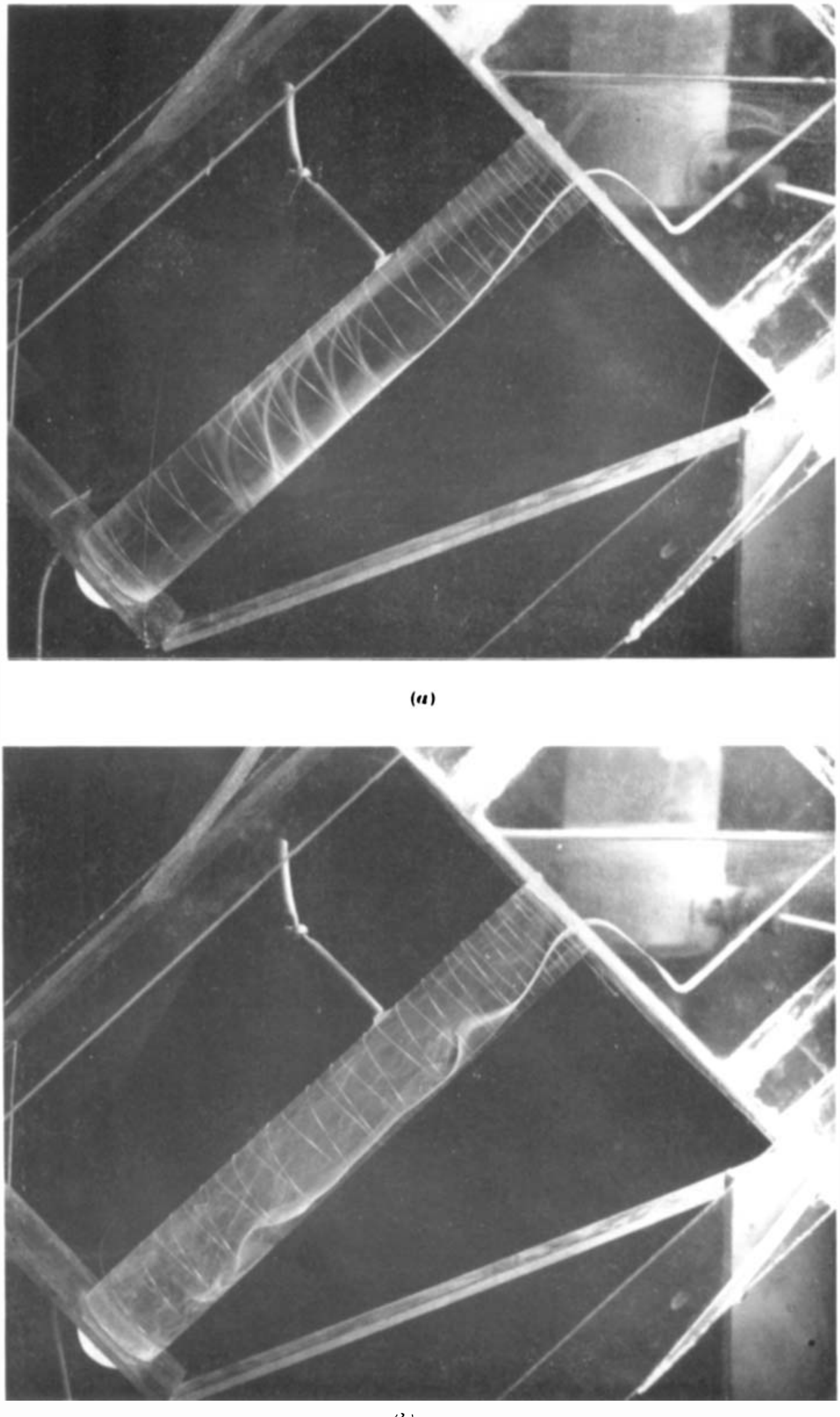
On this basis the rounded orifice at low Rayleigh numbers probably permits relatively easier access of the cold fluid. This is because more of the curtain of hot fluid is exposed, and hence larger entry holes are available to the incoming fluid.



(a)

(b)

FIGURE 2. Mode of penetration of entry fluid under turbulent conditions.


(*a*)

(*b*)

FIGURE 3. Effect of tilting on orifice instability.

It therefore seems likely that the higher heat-transfer rates with the rounded orifice are due to better circulation, the gain being 25 % at $Ra_L = 10^{8\cdot5}$ over the sharp-edged orifice. At this stage the latter is more likely to worsen the orifice disturbance and thereby to promote flow reversal and turbulent mixing of the fluid streams to which Martin (1955) has drawn attention as being a prime cause of reduced circulation.

The surprisingly high value of 0·36 for the slope of the curve using the sharp-edged orifice would also seem indicative of a trend towards turbulence, although ostensibly the flow is laminar. In this sense the sharp-edged orifice might be regarded as a beneficial turbulence promoter if the existing trend at the higher Rayleigh numbers could be maintained, where its effect on the heat-transfer performance is in figure 4 superior to that of the rounded orifice. This would require that the mixing zone remain confined to the region of the orifice, so that while some degree of turbulence is imparted to the fluid circulating below this region, viscous forces are still sufficiently large to prevent any extension of the mixing zone.

That this was so in the tests was proved by conducting longitudinal traverses along the tube axis, which showed that there was no appreciable rise in temperature of the cool fluid below the orifice region. An indirect proof of the existence of a more pronounced mixing zone with the sharp-edged orifice was provided by the need to reduce the power input to the lower heater when this orifice was substituted for the $\frac{7}{8}$ in. rounded orifice, if a uniform wall temperature were to be maintained. Martin (1955) has shown this to be a most sensitive indication of the onset of mixing. It thus seems that for $Ra_L > 10^{10\cdot1}$, the degree of turbulence imparted to the boundary-layer fluid below the mixing zone, and produced by the sharp-edged orifice, improves the heat transfer to an extent which more than offsets the decrease caused by reduced circulation.

(*b*) *Turbulent flow*

As in the case of laminar flow, the preliminary test using a $\frac{1}{4}$ in. rounded orifice, with water as the convective fluid, gave results which were in good agreement with those obtained for water by Martin (1955) in a similar tube. Longitudinal temperature traverses of the tube axis made during the test showed that as the Rayleigh number increased, the region of increasing temperature of the cool fluid, i.e. the mixing region, spread from the orifice region towards the closed end. The process was complete at $Ra_L = 10^{11\cdot5}$, when the fully-mixed régime substantially filled the cavity. At this stage there was no further decrease in the percentage of heat supplied by the lower heater. During development, the temperature rise across the mixing region was found to vary with the depth of the régime raised to a power of approximately 2·5. The evidence of the flow-visualization experiments, that mixing begins at the orifice, and in a fluid of sufficiently low viscosity spreads down the tube to establish the fully-mixed flow régime, is thus adequately confirmed.

The results of water tests using the other two orifices are displayed in figure 5. As with rape-seed oil, two curves, both of uniform slope, are presented, but because of the difference in viscosity-temperature characteristics, the tests cover a wider range of Rayleigh numbers. It will be seen that compared with figure 4, the positions of the curves for the two orifices are almost interchanged. The confirmatory test results using the $\frac{1}{4}$ in. rounded orifice (not shown) again occupy an intermediate position. The sharp-edged orifice gives generally better heat transfer than the $\frac{7}{8}$ in. rounded orifice over the range covered. But because the curve for the latter has a slope of 0·32, whereas that of the former is 0·29, the curves rapidly approach each other at the high Rayleigh numbers. It is possible that there would be little to choose between the two orifices for values of Ra_L greater than about $10^{11\cdot5}$.

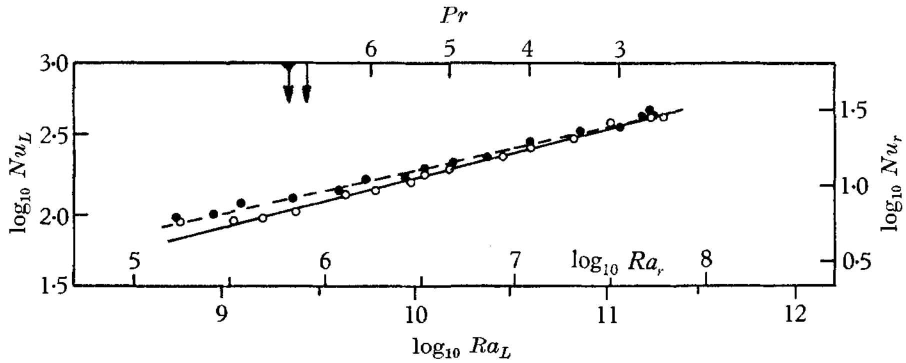


FIGURE 5. Experimental heat-transfer rates in turbulent flow for rounded and sharp-edged orifices. ○, Results for $\frac{7}{8}$ in. radius orifice; slope = 0·32. ●, Results for sharp-edges orifice; slope = 0·29. →, Onset of fully-mixed turbulent flow for $\frac{7}{8}$ in. radius orifice. ►→, Onset of fully-mixed turbulent flow for sharp-edged orifice.

It is again felt that the explanation lies in the fact that the sharp-edged orifice is a better promoter of turbulence. Measurements of the distribution of transferred heat indicate the onset of fully mixed flow at $Ra_L = 10^{9\cdot3}$ for the sharp-edged orifice and $Ra_L = 10^{9\cdot4}$ for the rounded orifice. At lower Rayleigh numbers the sharp-edged orifice gives better heat transfer because it exercises greater influence on the predominant boundary-layer régime. This has already been found to be the circumstance in which best heat transfer is obtained, notwithstanding the adverse effect on the circulation caused by the shallow mixing region at the orifice.

But with the onset, development and ultimate establishment of fully-mixed flow throughout the tube with a fluid of low viscosity, the orifice shape merely determining the Rayleigh number at which each stage occurs, the effect on the circulation becomes increasingly severe. This is, of course, because impeding of the flow is no longer confined to the orifice region. Thus the initial advantage of using the sharp-edged orifice is increasingly offset during the development of fully-mixed flow by the fact that it produces more of the régime at any given Rayleigh number. When this is large enough to ensure fully-mixed flow throughout the tube, whatever the shape of the orifice, the heat transfer may also be substantially independent of orifice shape, as figure 5 suggests.

In practical applications, where turbulent conditions are likely to predominate, it therefore seems that, of the orifice shapes investigated, the sharp-edge type is to be preferred. Not only is this easier to manufacture, but it covers the possibility of an extended range of working conditions over part of which the flow might be of the boundary-layer type. A sharp-edged orifice would then give higher heat transfer. This may well be the case in gas turbine rotor-blade passages. Martin (1959) has suggested that Coriolis forces may be insufficient to eliminate completely the mixing effect in the manner shown by the flow-visualization experiments to be achieved by tilting the tube.

Conclusions

Visualization of the flow in the orifice region of the free convection open thermosyphon has shown that for a tube whose length is great in relation to its radius, cool fluid always attempts to enter the tube from layers near the reservoir floor. The resulting collision with the hot fluid simultaneously discharged in the form of a necking annulus produces flow instability. Cool fluid which is not admitted is carried up with the rising hot stream.

There are two principal modes by which the cold fluid penetrates the heated annular curtain. In the laminar impeded flows the curtain parts to form holes through which some of the cold fluid can pass. Otherwise, for both laminar and turbulent boundary-layer flow the mode of penetration is as follows. The cool fluid gains entry by mixing with part of the heated annulus and carries it back into the tube. The mixing may be confined to a shallow region near the orifice if viscous forces are sufficient to maintain laminar flow elsewhere. Otherwise, under turbulent conditions, mixing may extend throughout the tube, though a boundary-layer régime can be restored by tilting the tube so as to eliminate the orifice instability.

These observations are useful in explaining the effects of changes in the shape of the entry orifice on measured heat-transfer rates. For laminar flow in a vertical tube a rounded orifice gave better heat transfer at low Rayleigh numbers than a sharp-edged orifice. The latter, it is suggested, causes mixing at the orifice, corresponding more closely to the second mode of penetration, and thereby reduces the circulation. At large Rayleigh numbers the sharp-edged orifice gave higher heat-transfer rates because the degree of turbulence introduced by the mixing region into the hot laminar stream more than offset the reduced circulation.

In turbulent flow the sharp-edged orifice was also initially superior to the rounded orifices in promoting heat transfer, but due to the subsequent development of mixing throughout the tube at high Rayleigh numbers, the heat transfer ultimately became virtually independent of orifice shape. In the light of the investigation it is suggested that sharp-edged orifices are preferable to the rounded variety in practical applications of the open thermosyphon. However, this does not preclude the possibility that other shapes, such as the serrated or saw-toothed type of orifice, might, through the promotion of turbulence to the maximum extent, yield even better overall heat transfer.

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